GENERAL TOPOLOGY
EXERCISES FOR SESSION 3 (TUE 5.3)

Exercise 1. *Let* $A \subset X$ *and suppose that there is a sequence of points* $(x_n) \subset A$ *converging to some* $a \in X$*. Show that* $a \in \bar{A}$*.*

Exercise 2. *Suppose that* $\mathbb{R}$ *is equipped with the cofinite topology (*A *is open if* $A = \emptyset$ *or* A^c *is finite). Find all limit points of the sequence* $1, 2, 3, \ldots$*.*

Exercise 3. *Let* X *be Hausdorff and* $A \subset X$*. Prove that* A *is Hausdorff in the relative topology.*

Exercise 4. *Let* $\mathbb{R}$ *be equipped with the topology given by the basis* $\{(-R, R) : R > 0\}$*. Show that a sequence has a limit if and only if it is bounded (i.e.* $|x_n| \leq M$ *for some* $M > 0$ *and all* n*).*

Exercise 5. *Show that in a* T_1*-space* X*, a point* x *is an accumulation point of a set* A *if and only if every neighbourhood of* x *contains infinitely many points in* A*.*

Exercise 6. *Let* $A \subset X$ *and let* $(x_n) \subset A$ *be a converging sequence in the relative topology on* A*. Show that* (x_n) *also converges in the topology of* X*.*

For which sets A *is the reverse of this statement always true, i.e. if* $(x_n) \subset A$ *converges in* X*, then it converges in the subspace* A *with the relative topology?*

Exercise 7. *Prove the second part of Theorem 2.4 in the lecture notes: If* X *is first-countable,* $A \subset X$ *and* $a \in \bar{A}$*, then there is a sequence in* A *converging to* a*.*