# GENERAL TOPOLOGY
# EXERCISES FOR SESSION 8 (WED 20.3)

**Exercise 1.** *Verify that the one-point compactification $X^*$ defined in Definition 3.3 is actually compact.*

**Exercise 2.** *Show that a closed subspace of a normal space is normal.*

**Exercise 3.** *Show that every compact Hausdorff space is normal. (Hint: Theorem 3.4)*

**Exercise 4.** *Show that if $X$ is normal and $F, G \subset X$ are closed, then there are open neighbourhoods $U, V$ of $F, G$ respectively such that $\bar{U} \cap \bar{V} = \emptyset$.*

**Exercise 5.** *Show that every locally compact Hausdorff space is regular. You can follow this scheme:*

- *Pick a closed set $F \subset X$ and $x \notin F$.*
- *Consider a neighbourhood $V$ of $x$ such that $\bar{V}$ is compact.*
- *Handle the situation inside $\bar{V}$ first.*
- *Then you can solve the general situation easily.*

**Exercise 6.** *Show that the one-point compactification of $\mathbb{Z}_+$ is homeomorphic with the subspace $\{0\} \cup \{\frac{1}{n} : n \geq 1\}$ of $\mathbb{R}$.*

**Exercise 7.** *A space $X$ is said to be* ***completely normal*** *if every subspace of $X$ is normal. Prove that $X$ is completely normal if and only if for every pair of sets $A, B \subset X$ with $\bar{A} \cap B = \emptyset = A \cap \bar{B}$, there exist disjoint neighbourhoods of $A$ and $B$.*

**Exercise 8.** *Are metrizable spaces completely normal?*