

GENERAL TOPOLOGY

HOMEWORK FOR WEEK 1

DEADLINE: MON 4.3, 23:59

Exercise 1. Let A be an open set in a topological space X . Prove that $\text{int } \partial A = \emptyset$.

Exercise 2. Find the mistake in the following reasoning by ChatGPT. Note that the statement it is trying to prove is not true in the first place.

(You can also compare this to the fourth point in Theorem 1.3 in the lecture notes - the statement of Theorem 1.3 is not always true when we have an infinite amount of sets A_i .)



You

Let A_i be some sets in a topological space X . Prove that the closure of their union is contained in the union of their closures.



ChatGPT

To prove that the closure of the union of sets A_i is contained in the union of their closures, we can proceed by showing that any accumulation point of the union of sets A_i is also an accumulation point of the union of their closures.

Let x be an accumulation point of $\cup A_i$. This means that every neighborhood of x contains a point of $\cup A_i$ different from x .

Since x is an accumulation point of $\cup A_i$, there exists an index j such that x is an accumulation point of A_j . This means that every neighborhood of x contains a point of A_j different from x .

Since x is an accumulation point of A_j , it follows that x belongs to the closure of A_j , denoted $\overline{A_j}$. Therefore, x is also a point in the union of closures of sets A_i , i.e., $x \in \cup \overline{A_i}$.

Thus, we have shown that any accumulation point of $\cup A_i$ is also a point of $\cup \overline{A_i}$, which implies that the closure of $\cup A_i$ is contained in the union of their closures.

Hence, we have proved that:

$$\overline{\cup A_i} \subseteq \cup \overline{A_i}$$