GENERAL TOPOLOGY
EXERCISES FOR SESSION 7 (TUE 18.3)

Exercise 1. *Prove that if* $A \subset X$ *is connected, then* $\bar{A}$ *is connected.*

Exercise 2. *Let* A *be connected. Is* $\mathrm{int}(A)$ *necessarily connected?*

Exercise 3. *Prove the* ***Border Crossing Theorem****: Let* $E \subset X$ *be connected and* $A \subset X$*. Prove that if* E *intersects* A *and* A^{c}*, then it intersects* ∂A*.*

Exercise 4. *Suppose that in the space* X *for every pair of points* $x, y \in X$ *there exists a connected set* $E(x, y)$ *which contains both* x *and* y*. Prove that* X *is connected.*

Conclude that every path connected space is connected.

Exercise 5. *Prove that a space* X *is locally connected if and only if the components of every open subset* $U \subset X$ *are also open sets.*

Exercise 6. *Prove that none of the sets* $[0, 1]$*,* $[0, 1)$ *and* $(0, 1)$ *are homeomorphic. (Hint: How can you use connectedness?)*

Exercise 7. *Show that if a metric space* X *is connected and has more than one point, it is uncountable. (Hint: Find a surjection from* X *to an interval in* $\mathbb{R}$*.)*

Exercise 8. *Let* $A \subset X$ *and suppose that both* A *and* X *are connected. Show that if* U *and* V *form a separation of* $X \setminus A$*, then both* $A \cup U$ *and* $A \cup V$ *are connected.*

Exercise 9. *Let* $A, B \subset X$ *be open sets such that* $A \cup B$ *and* $A \cap B$ *are connected. Show that* A *and* B *are connected.*

Exercise 10. *Show that if* $A \subset \mathbb{R}^2$ *is countable, then* $\mathbb{R}^2 \setminus A$ *is path connected.*