GENERAL TOPOLOGY
EXERCISES FOR SESSION 9 (TUE 25.3)

Exercise 1. *Show that a subset $S \subset X$ is dense if and only if $\bar{S} = X$.*

Exercise 2. *Prove that a second-countable space is Lindelöf.*

Exercise 3. *Let $f, g : X \to Y$ be continuous functions with Y being Hausdorff. Suppose that $f(s) = g(s)$ for every point s in a dense set $S \subset X$. Show that $f(x) = g(x)$ for all $x \in X$.*

The next few exercises concern the **lower limit topology** on $\mathbb{R}$. This topology $\mathcal{T}_\ell$ is generated by the basis of half-open intervals $[a, b) \subset \mathbb{R}$, $a < b$. The resulting topological space is denoted by $\mathbb{R}_\ell$, and is also sometimes called the *Sorgenfrey line.*

Exercise 4. *Find the connected components of $\mathbb{R}_\ell$.*

Exercise 5. *Check what separation axioms $\mathbb{R}_\ell$ satisfies.*

Exercise 6. *Does the sequence $x_n = 1 - \frac{1}{n}$ have a cluster point in $\mathbb{R}_\ell$?*

Exercise 7. *Is $\mathbb{R}_\ell$ locally compact?*

Exercise 8. *Prove that $\mathbb{R}_\ell$ is Lindelöf. (difficult)*

Exercise 9. *Let $f_j : X \to Y_j$ be a countable collection of functions to spaces Y_j which are second-countable. Prove that the topology on X induced by the f_j's is second-countable.*

Exercise 10. *Let X be second-countable. Show that if $A \subset X$ is uncountable, then $\operatorname{acc}(A) \cap A$ is uncountable.*