GENERAL TOPOLOGY
EXERCISES FOR SESSION 12 (WED 2.4)

Exercise 1. *Let* $X = \prod_{i \in I} X_i$ *with* $X_i \neq \emptyset$ *for all* i. *Suppose that* X *is compact. Prove that* X_i *is compact for all* $i \in I$.

Exercise 2. *Let* X *be a topological vector space. Show that if* $F, G \subset X$ *are compact, then* $F + G$ *is compact.*

Exercise 3. *A space is called* σ***-compact*** *if it is the countable union of compact sets. Show that a* σ*-compact space is Lindelöf.*

Exercise 4. *Prove that a locally compact Hausdorff space which is Lindelöf is also* σ*-compact.*

Exercise 5. *Show that if* X *is Lindelöf and* Y *is compact, then* $X \times Y$ *is Lindelöf.*

Exercise 6. *Show that if* X *and* Y *are locally compact, then* $X \times Y$ *is locally compact.*

Exercise 7. *Let* $[0,1]^{\mathbb{N}}$ *be the Hilbert cube. Points in this space can be interpreted also as sequences* $(x_0, x_1, \ldots)$ *with* $x_i \in [0,1]$ *for all* $i \in \mathbb{N}$. *If* $A \subset [0,1]^{\mathbb{N}}$ *denotes the subset of all converging sequences, is* A *closed in the product topology?*

Exercise 8. *Suppose that* X *is compact. Prove that for every other topological space* Y *the projection* $\mathrm{proj}_Y : X \times Y \to Y$ *is a closed map.*

Note: The converse of this statement is also true, but it is very challenging to prove (feel free to try).